Composition of Transformations

The symbol for a composition of transformations (or functions) is an open circle.

A notation such as $T_{1,5} \circ r_{y=x}$ is read as: "a translation of $(x, y) \rightarrow (x + 1, y + 5)$ after a reflection in the line $y = x$".

You may also see the notation written as $T_{1,5} \left(r_{y=x}\right)$.

**BEWARE** This process must be done from right to left (\[ \rightarrow \])!!

Composition of transformations is **not commutative**.
As the graphs below show, if the transformation is read from left to right, the result will NOT be the same as reading from right to left.

$$T_{1,5} \circ r_{y=x} \neq r_{y=x} \circ T_{1,5}$$

A composition of any **translation** or **rotation** can be expressed as the composition of two reflections.

*Example:* $r_{x\text{-axis}} \circ r_{y\text{-axis}} = R_{180^\circ}$
A composition of **reflections over two parallel lines** is equivalent to a translation.

**Example:** Given \( a \parallel b \), and pre-image \( \triangle ABC \), where parallel lines are vertical.

\[
(r_{\text{line}_b} \circ r_{\text{line}_a})(\triangle ABC) = \triangle A''B''C''
\]

The composition of **reflections over two intersecting lines** is equivalent to a rotation.

**Example:** Given two lines, \( a \) and \( b \), intersecting at point \( P \), and pre-image \( \triangle ABC \).

\[
(r_{\text{line}_b} \circ r_{\text{line}_a})(\triangle ABC) = \triangle A''B''C''
\]

A **glide reflection** is the composition of a **reflection** and a **translation**, where the line of reflection, \( m \), is parallel to the directional vector line, \( v \), of the translation.

**Example:** \((r_m \circ T_v)(\triangle ABC) = \triangle A''B''C''\)

The composition of two **rotations** from the same center, is a **rotation** whose degree of rotation equals the sum of the degree rotations of the two initial rotations.

**Example:** \((R_{C,45^\circ} \circ R_{C,90^\circ})(P) = R_{C,135^\circ}(P)\)
1. a) Given \( \triangle ABC \) with coordinates \( A(-1, -2) \), \( B(0, 4) \), and \( C(3, -1) \)

   b) Graph and label \( \triangle A'B'C' \) the image of \( \triangle ABC \) after a reflection in the line \( y = -x \).

2. a) Given \( \triangle ABC \) with coordinates \( A(-1, -2) \), \( B(0, 4) \), and \( C(3, -1) \)

   b) Graph and label \( \triangle A'B'C' \) the image of \( \triangle ABC \) after a reflection in the \( y \)-axis.
3. a) Given $\triangle ABC$ with coordinates $A(2, 3)$, $B(0, 6)$ and $C(2, 6)$
   b) Graph and label $\triangle A'B'C'$ the image of $\triangle ABC$ after a translation $T_{4,-3}$
   c) Graph and label $\triangle A''B''C''$ the image of $\triangle A'B'C'$ after a reflection in the $y = x$.

4. a) Given $\triangle ABC$ with coordinates $A(0, 9)$, $B(-3, 0)$ and $C(-6, 9)$
   b) Graph and label $\triangle A'B'C'$ the image of $\triangle ABC$ after a reflection through the origin
   c) Graph and label $\triangle A''B''C''$ the image of $\triangle A'B'C'$ after a dilation of $\frac{1}{3}$.
   d) Graph and label $\triangle A'''B'''C'''$ the image of $\triangle A''B''C''$ after a translation $T_{5,4}$.
5. a) Given $\triangle ABC$ with coordinates $A(2, 3)$, $B(0, 6)$ and $C(2, 6)$
   b) Graph and label $\triangle A'B'C'$ the image of $\triangle ABC$ after a reflection in the y-axis.
   c) Graph and label $\triangle A''B''C''$ the image of $\triangle A'B'C'$ after a reflection $y = x$
   d) Graph and label $\triangle A'''B'''C'''$ the image of $\triangle A''B''C''$ after a rotation clockwise about the origin $90^\circ$

6. a) Given $\triangle ABC$ with coordinates $A(1, 2)$, $B(0, 5)$ and $C(5, 4)$
   b) Graph and label $\triangle A'B'C'$ the image of $\triangle ABC$ after $(x, y) \rightarrow (x - 6, y + 3)$
   c) Graph and label $\triangle A''B''C''$ the image of $\triangle A'B'C'$ after a reflection x-axis
   d) Graph and label $\triangle A'''B'''C'''$ the image of $\triangle A''B''C''$ after a reflection in the origin.
7. a) Given $\triangle ABC$ with coordinates $A(1, 2)$, $B(4, -2)$ and $C(5, 4)$ 
    b) Graph and label $\triangle A'B'C'$ the image of $\triangle ABC$ after a reflection x-axis 
    c) Graph and label $\triangle A''B''C''$ the image of $\triangle A'B'C'$ after a reflection in the origin. 
    d) Graph and label $\triangle A'''B'''C'''$ the image of $\triangle A''B''C''$ after a dilation of 2

8. a) Given $\triangle ABC$ with coordinates $A(-1, 2)$, $B(5, 2)$ and $C(3, 4)$ 
    b) Graph and label $\triangle A'B'C'$ the image of $\triangle ABC$ after the composition $R_{90} \circ r_{x-axis}$ 
    c) Graph and label $\triangle A''B''C''$ the image of $\triangle ABC$ after $D_2$
9. a) Given \( \triangle ABC \) with coordinates \( A(4, 2), B(8, 2) \) and \( C(2, 6) \)
   b) Graph and label \( \triangle A'B'C' \) the image of \( \triangle ABC \) after the composition \( R_{180} \circ T_{2, 4} \)
   c) Graph and label \( \triangle A''B''C'' \) the image of \( \triangle ABC \) after \( D_{1/2} \)

10. a) Given \( \triangle ABC \) with coordinates \( A(1, 0), B(6, 3) \) and \( C(4, 5) \)
    b) Graph and label \( \triangle A'B'C' \) the image of \( \triangle ABC \) after the composition \( D_2 \circ r_0 \)
11. a) Given $\triangle ABC$ with coordinates $A(-1, 4)$, $B(3, 7)$ and $C(5, 1)$

   b) Graph and label $\triangle A'B'C'$ the image of $\triangle ABC$ after the composition $R_{90^\circ} \circ r_{x\text{-axis}}$

![Graph](image1)

12. a) Given $\triangle ABC$ with coordinates $A(2, -3)$, $B(1, 1)$ and $C(-2, -1)$

   b) Graph and label $\triangle A'B'C'$ the image of $\triangle ABC$ after the composition $D_{2/3} \circ D_3$

   c) Graph and label $\triangle A''B''C''$ the image of $\triangle ABC$ after $T_{8, -4}$

![Graph](image2)
13. a) Given \( \triangle ABC \) with coordinates \( A(-1, 3) \), \( B(3, 7) \) and \( C(0, -6) \)  

b) Graph and label \( \triangle A'B'C' \) the image of \( \triangle ABC \) after the composition \( r_{y-axis} \circ r_y = x \) 

c) Graph and label \( \triangle A''B''C'' \) the image of \( \triangle A'B'C' \) after \( T_{0.5} \)

14. a) Given \( \triangle ABC \) with coordinates \( A(-5, 1) \), \( B(-2, 5) \) and \( C(-7, 4) \)  

b) Graph and label \( \triangle A'B'C' \) the image of \( \triangle ABC \) after the composition \( r_0 \circ r_y \times x \) 

c) Graph and label \( \triangle A''B''C'' \) the image of \( \triangle A'B'C' \) after \( T_{2,7} \)
15. a) Given \( \triangle ABC \) with coordinates \( A(3, 4) \), \( B(1, 7) \) and \( C(3, 7) \)

b) Graph and label \( \triangle A'B'C' \) the image of \( \triangle ABC \) after the composition \( r_y = x \circ r_{y-axis} \)

c) Graph and label \( \triangle A''B''C'' \) the image of \( \triangle A'B'C' \) after \( T_{5,-1} \)

16. a) Given \( \triangle ABC \) with coordinates \( A(-1, 3) \), \( B(-6, 5) \) and \( C(-4, 7) \)

b) Graph and label \( \triangle A'B'C' \) the image of \( \triangle ABC \) after the composition \( r_y = x \circ r_{x-axis} \)

c) Graph and label \( \triangle A''B''C'' \) the image of \( \triangle ABC \) after \( T_{8,3} \)
17. a) Given \( \triangle ABC \) with coordinates \( A(1, 0), B(7, 4) \) and \( C(5, 7) \)

b) Graph and label \( \triangle A'B'C' \) the image of \( \triangle ABC \) after the composition \( T_{1,5} \circ r_0 \)

c) Graph and label \( \triangle A''B''C'' \) the image of \( \triangle ABC \) after \( r_{y-axis} \)

\[
\begin{align*}
1. & \quad T_{(-3,5)} \circ R_{y-axis} (\triangle ABC) \\
& \quad A' = (__,__) \quad A'' = (__,__) \\
& \quad B' = (__,__) \quad B'' = (__,__) \\
& \quad C' = (__,__) \quad C'' = (__,__) \\
2. & \quad R_{y-axis} \circ T_{(-3,5)} (\triangle ABC) \\
& \quad A' = (__,__) \quad A'' = (__,__) \\
& \quad B' = (__,__) \quad B'' = (__,__) \\
& \quad C' = (__,__) \quad C'' = (__,__) \\
\end{align*}
\]

3. Looking at #1 & 2 above, does doing the transformations in a different order matter? Explain why?
4. $R_{x \text{axis}} \circ R_{O,90^\circ} (\triangle ABC)$

A' = (__,__) A'' = (__,__)

B' = (__,__) B'' = (__,__)

C' = (__,__) C'' = (__,__)

5. $R_{O,90^\circ} \circ R_{x \text{axis}} (\triangle ABC)$

A' = (__,__) A'' = (__,__)

B' = (__,__) B'' = (__,__)

C' = (__,__) C'' = (__,__)

6. Looking at #4 & 5 above, does doing the transformations in a different order matter? Explain why?
7. \( R_{x=-2} \circ R_{x=-3}(BC) \)

\[ B' = (__,__) \quad B'' = (__,__) \]

\[ C' = (__,__) \quad C'' = (__,__) \]

Circle the resultant transformation from \( \overline{BC} \) to \( \overline{B''C''} \)?

- Rotation
- Reflection
- Translation

Describe the transformation. Specifically its direction and distance from B to B'' and from C to C''.

What does the distance of the transformation from B to B'' have to do with the distance between the two parallel lines?

8. \( R_{x=-5} \circ R_{x=-2}(BC) \)

\[ B' = (__,__) \quad B'' = (__,__) \]

\[ C' = (__,__) \quad C'' = (__,__) \]

Circle the resultant transformation from \( \overline{BC} \) to \( \overline{B''C''} \)?

- Rotation
- Reflection
- Translation

Describe the transformation. Specifically its direction and distance from B to B'' and from C to C''.

What does the distance of the transformation from B to B'' have to do with the distance between the two parallel lines?
9. \( R_{x=2} \circ R_{x=-4}(\Delta ABC) \)
\( A' = (__,__) \quad A'' = (__,__) \)
\( B' = (__,__) \quad B'' = (__,__) \)

Circle the resultant transformation from \( \Delta ABC \) to \( \Delta A''B''C''' \)?

Rotation \quad Reflection \quad Translation

Describe the transformation. Specifically its direction and distance from B to B'' and from C to C''.

What does the distance of the transformation from B to B'' have to do with the distance between the two parallel lines?

10. \( R_{x=2} \circ R_{x=-4}(\Delta ABC) \)
\( A' = (__,__) \quad A'' = (__,__) \)
\( B' = (__,__) \quad B'' = (__,__) \)

Circle the resultant transformation from \( \Delta ABC \) to \( \Delta A''B''C''' \)?

Rotation \quad Reflection \quad Translation

Describe the transformation. Specifically its direction and distance from B to B'' and from C to C''.

What does the distance of the transformation from B to B'' have to do with the distance between the two parallel lines?
11. $R_{y=4} \circ R_{y=1}(\overline{AB})$

\[ \begin{align*}
A' &= (__,__) & A'' &= (__,__) \\
B' &= (__,__) & B'' &= (__,__) 
\end{align*} \]

Circle the resultant transformation from $\overline{AB}$ to $\overline{A''B''}$?

rotation  reflection  translation

Describe the transformation. Specifically its direction and distance from $A$ to $A''$ and from $B$ to $B''$.

What does the distance of the transformation from $A$ to $A''$ have to do with the distance between the two parallel lines?

12. $R_{y=1} \circ R_{y=4}(\overline{AB})$

\[ \begin{align*}
A' &= (__,__) & A'' &= (__,__) \\
B' &= (__,__) & B'' &= (__,__) 
\end{align*} \]

Circle the resultant transformation from $\overline{AB}$ to $\overline{A''B''}$?

rotation  reflection  translation

Describe the transformation. Specifically its direction and distance from $A$ to $A''$ and from $B$ to $B''$.

What does the distance of the transformation from $A$ to $A''$ have to do with the distance between the two parallel lines?
13. \( R_{y = -3} \circ R_{y = -4}(\Delta ABC) \)
A' = (__,__)  A'' = (__,__)
B' = (__,__)  B'' = (__,__)
C' = (__,__)  C'' = (__,__)
Circle the resultant transformation

14. \( R_{y = -3} \circ R_{y = -5}(\Delta ABC) \)
A' = (__,__)  A'' = (__,__)
B' = (__,__)  B'' = (__,__)
C' = (__,__)  C'' = (__,__)
Circle the resultant transformation

\( \Delta ABC \) to \( \Delta A''B''C'''? \)

<table>
<thead>
<tr>
<th>Rotation</th>
<th>Reflection</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Describe the transformation. Specifically its direction and distance from A to A'' and from B to B''.

What does the distance of the transformation from A to A'' have to do with the distance between the two parallel lines?

15. **Summary of relationships found in previous exercises.**

a) Summarize why the order matters for two transformations.

b) Would the order matter for two translations? Why or why not?

c) A double reflection over two parallel lines results in a ________________________________.

d) The distance of the translation is exactly ________________ the distance between the parallel lines.

e) The direction of the translation depends on ________________________________.
16. Complete the following
a) If you wanted to translate a shape to the right 10 units, you could reflect over \( x = 3 \) and then \( x = \) ______.

b) If you wanted to translate a shape to the left 4 units, you could reflect over \( x = 5 \) and then \( x = \) ______.

c) If you wanted to translate a shape up 6 units, you could reflect over \( y = -1 \) and then \( y = \) ______.

d) If you wanted to translate a shape down 1 units, you could reflect over \( y = 8 \) and then \( y = \) ______.

e) If you wanted to translate a shape to the right 6 units, you could reflect over \( x = \) ______ and then \( x = -4 \).

f) If you wanted to translate a shape to the left 12 units, you could reflect over \( x = \) ______ and then \( x = 1 \).

g) If you wanted to translate a shape up 4 units, you could reflect over \( y = \) ______ and then \( y = 2 \).

h) If you wanted to translate a shape down 7 units, you could reflect over \( y = \) ______ and then \( y = -2.5 \).

17. - 19. Determine the missing line of reflection. Draw it in and complete the composite statement.

17. \( R_y = \) ______ \( \circ \) \( R_y = 4(AB) \)

18. \( R_x = \) ______ \( \circ \) \( R_x = -1(AB) \)

19. \( R_x = -2 \circ R_x = \) ______ \( (AB) \)
1. $R_y \circ R_x (\Delta ABC)$

$A' = (__,__) \quad A'' = (__,__)$

$B' = (__,__) \quad B'' = (__,__)$

$C' = (__,__) \quad C'' = (__,__)$

Circle the resultant transformation from $\Delta ABC$ to $\Delta A''B''C''$?

Rotation  Reflection  Translation

How did you recognize which transformation mapped $A$ to $A''$, $B$ to $B''$ and $C$ to $C''$?

What does the angle of the transformation $\angle AOA''$ have to do with the angle between the two intersecting lines?

2. $R_x \circ R_y (\Delta ABC)$

$A' = (__,__) \quad A'' = (__,__)$

$B' = (__,__) \quad B'' = (__,__)$

$C' = (__,__) \quad C'' = (__,__)$

Circle the resultant transformation from $\Delta ABC$ to $\Delta A''B''C''$?

Rotation  Reflection  Translation

How did you recognize which transformation mapped $A$ to $A''$, $B$ to $B''$ and $C$ to $C''$?

What does the angle of the transformation $\angle AOA''$ have to do with the angle between the two intersecting lines?
3. \( R_y \circ R_x(\Delta ABC) \)
A' = (__,__)  A'' =(__,__)
B' = (__,__)  B'' =(__,__)
C' = (__,__)  C'' =(__,__)

Circle the resultant transformation from \( \Delta ABC \) to \( \Delta A''B''C'' \)?
Rotation  Reflection  Translation

How did you recognize which transformation mapped A to A'', B to B'' and C to C'''?

What does the angle of the transformation \( \angle AOA'' \) have to do with the angle between the two intersecting lines?

4. \( R_x \circ R_y(\Delta ABC) \)
A' = (__,__)  A'' =(__,__)
B' = (__,__)  B'' =(__,__)
C' = (__,__)  C'' =(__,__)

Circle the resultant transformation from \( \Delta ABC \) to \( \Delta A''B''C'' \)?
Rotation  Reflection  Translation

How did you recognize which transformation mapped A to A'', B to B'' and C to C'''?

What does the angle of the transformation \( \angle AOA'' \) have to do with the angle between the two intersecting lines?
5. $R_{y \rightarrow x} \circ R_{x \rightarrow y}(\triangle ABC)$

$A' = (__,__)$  $A'' = (__,__)$
$B' = (__,__)$  $B'' = (__,__)$
$C' = (__,__)$  $C'' = (__,__)$

Circle the resultant transformation from $\triangle ABC$ to $\triangle A''B''C''$?

Rotation  Reflection  Translation

How did you recognize which transformation mapped $A$ to $A''$, $B$ to $B''$ and $C$ to $C'''$?

What does the angle of the transformation $\angle AOA''$ have to do with the angle between the two intersecting lines?

6. $R_{x \rightarrow y} \circ R_{y \rightarrow x}(\triangle ABC)$

$A' = (__,__)$  $A'' = (__,__)$
$B' = (__,__)$  $B'' = (__,__)$
$C' = (__,__)$  $C'' = (__,__)$

Circle the resultant transformation from $\triangle ABC$ to $\triangle A''B''C''$?

Rotation  Reflection  Translation

How did you recognize which transformation mapped $A$ to $A''$, $B$ to $B''$ and $C$ to $C'''$?

What does the angle of the transformation $\angle AOA''$ have to do with the angle between the two intersecting lines?

2. Summary of relationships found in previous exercises.

a) A double reflection over two intersecting lines results in a __________________________.

b) The angle of rotation of the double reflection over intersecting lines is exactly _______________ the angle between the intersecting lines.

c) The direction of the rotation depends on ________________________________.
3. Complete the following

a) If you wanted to rotate a shape by 90° by double reflecting it over two intersecting lines, the angle between the two intersecting lines would need to be ________.

b) If you wanted to rotate a shape by 110° by double reflecting it over two intersecting lines, the angle between the two intersecting lines would need to be ________.

c) If you wanted to rotate a shape by 24° by double reflecting it over two intersecting lines, the angle between the two intersecting lines would need to be ________.

d) If you wanted to rotate a shape by -200° by double reflecting it over two intersecting lines, the angle between the two intersecting lines would need to be ________.

1. Jane claims that any two circles are always isometric because the shape never changes. Is she correct? YES or NO Explain your answer.
2. Determine if the pre-image and image are isometric and also which transformation produced the image.

<table>
<thead>
<tr>
<th>PRE-IMAGE</th>
<th>Circle Answer</th>
<th>Circle Answer</th>
<th>IMAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image 1" /></td>
<td>Isometry</td>
<td>Rotation</td>
<td><img src="image2.png" alt="Image 2" /></td>
</tr>
<tr>
<td><img src="image3.png" alt="Image 3" /></td>
<td>Not Isometry</td>
<td>Reflection</td>
<td></td>
</tr>
<tr>
<td><img src="image4.png" alt="Image 4" /></td>
<td>Rotation</td>
<td>Translation</td>
<td></td>
</tr>
<tr>
<td><img src="image5.png" alt="Image 5" /></td>
<td>Reflection</td>
<td>Dilation</td>
<td></td>
</tr>
<tr>
<td><img src="image6.png" alt="Image 6" /></td>
<td>Translation</td>
<td>Stretch</td>
<td></td>
</tr>
<tr>
<td><img src="image7.png" alt="Image 7" /></td>
<td>Other</td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td>PRE-IMAGE</td>
<td>Circle Answer</td>
<td>Circle Answer</td>
<td>IMAGE</td>
</tr>
<tr>
<td>-----------</td>
<td>---------------</td>
<td>---------------</td>
<td>-------</td>
</tr>
<tr>
<td></td>
<td>Isometry</td>
<td>Rotation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflection</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Translation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dilation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stretch</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not Isometry</td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Isometry</td>
<td>Rotation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflection</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Translation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dilation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stretch</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not Isometry</td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Isometry</td>
<td>Rotation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflection</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Translation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dilation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stretch</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not Isometry</td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Isometry</td>
<td>Rotation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflection</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Translation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dilation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stretch</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not Isometry</td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Isometry</td>
<td>Rotation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflection</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Translation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dilation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stretch</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not Isometry</td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Isometry</td>
<td>Rotation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflection</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Translation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dilation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stretch</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not Isometry</td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Isometry</td>
<td>Rotation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflection</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Translation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dilation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stretch</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not Isometry</td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Isometry</td>
<td>Rotation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflection</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Translation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dilation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stretch</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not Isometry</td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Determine the coordinates of the image, plot the image and determine if it is an isometric transformation or not.

<table>
<thead>
<tr>
<th>PRE-IMAGE</th>
<th>Transformation</th>
<th>COORDINATES</th>
<th>PLOT THE IMAGE</th>
</tr>
</thead>
</table>
| a) Pre-Image Points | A (1, -4)  
B (2, -1)  
C (6, -4) | A' (______,______)  
B' (______,______)  
C' (______,______) | Isometry? Yes or No  
Transformation Type: |
| Coordinate Rule | (x, y) → (x - 5, y + 3) |  |
| b) Pre-Image Points | A (1, -4)  
B (2, -1)  
C (6, -4) | A' (______,______)  
B' (______,______)  
C' (______,______) | Isometry? Yes or No  
Transformation Type: |
| Coordinate Rule | (x, y) → (-x, y) |  |
| c) Pre-Image Points | A (1, -4)  
B (2, -1)  
C (6, -4) | A' (______,______)  
B' (______,______)  
C' (______,______) | Isometry? Yes or No  
Transformation Type: |
| Coordinate Rule | (x, y) → (y, -x) |  |
| d) Pre-Image Points | A (-5, -1)  
B (-4, 2)  
C (0, -1) | A' (______,______)  
B' (______,______)  
C' (______,______) | Isometry? Yes or No  
Transformation Type: |
| Coordinate Rule | (x, y) → (x, 3y) |  |
| e) Pre-Image Points | A (-2, -1)  
B (-1, 2)  
C (3, -1) | A' (______,______)  
B' (______,______)  
C' (______,______) | Isometry? Yes or No  
Transformation Type: |
| Coordinate Rule | (x, y) → (2x, 2y) |  |
| f) Pre-Image Points | A (1, -3)  
B (2, 0)  
C (6, -3) | A' (______,______)  
B' (______,______)  
C' (______,______) | Isometry? Yes or No  
Transformation Type: |
<p>| Coordinate Rule | (x, y) → (-x, -y) |  |</p>
<table>
<thead>
<tr>
<th>PRE-IMAGE</th>
<th>Transformation</th>
<th>COORDINATES</th>
<th>PLOT THE IMAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Pre-Image Points A (1, -4) B (2, -1) C (6, -4) Coordinate Rule ((x, y) \rightarrow (y, x)) Image Points A’ (____, <strong><strong>), B’ (</strong></strong>, <strong><strong>), C’ (</strong></strong>, ____), Isometry? Yes or No Transformation Type:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) Pre-Image Points A (-1, -2) B (0, 1) C (4, -2) Coordinate Rule ((x, y) \rightarrow (-2y, -x)) Image Points A’ (____, <strong><strong>), B’ (</strong></strong>, <strong><strong>), C’ (</strong></strong>, ____), Isometry? Yes or No Transformation Type:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) Pre-Image Points A (-3, 1) B (-2, 4) C (2, 1) Coordinate Rule ((x, y) \rightarrow (-y-2, x+3)) Image Points A’ (____, <strong><strong>), B’ (</strong></strong>, <strong><strong>), C’ (</strong></strong>, ____), Isometry? Yes or No Transformation Type:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) Pre-Image Points A (-5, -4) B (-3, 2) C (5, -4) Coordinate Rule ((x, y) \rightarrow (.5x, .5y)) Image Points A’ (____, <strong><strong>), B’ (</strong></strong>, <strong><strong>), C’ (</strong></strong>, ____), Isometry? Yes or No Transformation Type:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) Pre-Image Points A (0, 0) B (1, 3) C (5, 0) Coordinate Rule ((x, y) \rightarrow (-y, x)) Image Points A’ (____, <strong><strong>), B’ (</strong></strong>, <strong><strong>), C’ (</strong></strong>, ____), Isometry? Yes or No Transformation Type:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f) Pre-Image Points A (3, -2) B (2, 1) C (-2, -2) Coordinate Rule ((x, y) \rightarrow (x+y, y)) Image Points A’ (____, <strong><strong>), B’ (</strong></strong>, <strong><strong>), C’ (</strong></strong>, ____), Isometry? Yes or No Transformation Type:</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. If the letter $P$ is rotated 180 degrees, which is the resulting figure?
   1) $d$
   2) $a$
   3) $r$
   4) $b$

2. The accompanying diagram shows the starting position of the spinner on a board game.

3. If point $(5, 2)$ is rotated counter-clockwise 90° about the origin, its image will be point
   1) $(2, 5)$
   2) $(2, -5)$
   3) $(-2, 5)$
   4) $(-5, -2)$

4. What are the coordinates of $M'$, the image of $M(2, 4)$, after a counterclockwise rotation of 90° about the origin?
   1) $(-2, 4)$
   2) $(-2, -4)$
   3) $(-4, 2)$
   4) $(-4, -2)$

5. What is the image of $A(5, 2)$ under $R_{90}$?
   1) $(-5, 2)$

6. The transformation $R_{90}$ maps point $(5, 3)$ onto the point whose coordinates are
   1) $(5, -3)$
   2) $(3, -5)$
   3) $(3, 5)$
   4) $(-3, 5)$

7. What is the image of $A(5, 2)$ under $R_{90}$?
   1) $(-5, 2)$

8. What are the coordinates of the image of $P(-2, 5)$ after a clockwise rotation of 90° about the origin?
   ANS. 3
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2)</td>
<td>(5, -2)</td>
<td>1)</td>
</tr>
<tr>
<td>3)</td>
<td>(2, 5)</td>
<td>2)</td>
</tr>
<tr>
<td>4)</td>
<td>(-2, 5)</td>
<td>3)</td>
</tr>
<tr>
<td>4)</td>
<td>(5, 2)</td>
<td>4)</td>
</tr>
</tbody>
</table>

9. What are the coordinates of the image of (2, -5) after a counterclockwise rotation of 90° about the origin?

1) (-2, 5)
2) (2, 5)
3) (-5, -2)
4) (5, 2)

10. What is the image of the point (-3, 6) on rotation of 90° about the origin?

2. The point (-2, 1) is rotated 180° about the origin in a clockwise direction. What are the coordinates of its image?

11. What is the image of the point (2, 3) under a clockwise rotation of 90° R₋90 about the origin?

13. What is the image of R₋90(1, 2)?

14. Write the coordinates of P′, the image of P(5, 1) after a clockwise rotation of 180° about the origin.

15. What is the image of (5, 1) under a counterclockwise rotation of 90°?

16. The point (-3, 4) is rotated 180° about the origin in a counterclockwise direction. What are the coordinates of its image?

17. What is the image of (6, 5) under a counterclockwise rotation of 180°?

18. Point A is rotated 180° in a counterclockwise direction about the origin. If the coordinates of A are (-1, 3), what are the coordinates of A′, its image?

19. If point P(3, -2) is rotated 90° about the origin, what is the image of P?

20. The coordinates of the vertices of ΔRST are R(-2, 3), S(4, 4) and T(2, -2). Triangle R′S′T′ is the

21. The coordinates of the vertices of ΔABC are A(1, 2), B(-4, 3) and C(-3, -5). State the coordinates of
image of $\triangle RST$ after a rotation of $90^\circ$ about the origin. State the coordinates of the vertices of $\triangle R'S'T'$.  

$\triangle A'B'C'$, the image of $\triangle ABC$ after a rotation of $90^\circ$ about the origin.

<table>
<thead>
<tr>
<th>Question</th>
<th>Options</th>
</tr>
</thead>
</table>
| 1. Point $A$ is located at $(4, -7)$. The point is reflected in the $x$-axis. Its image is located at | 1) (-4, 7)  
2) (-4, -7)  
3) (4, 7)  
4) (7, -4) |
| 2. When the point $(2, 5)$ is reflected in the $x$-axis, what are the coordinates of its image? | 1) (-5, 2)  
2) (-2, 5)  
3) (2, 5)  
4) (5, 2) |
| 3. What is the image of point $(-3, 7)$ after a reflection in the $x$-axis? | 1) (3, 7)  
2) (-3, -7)  
3) (3, -7)  
4) (7, -3) |
| 4. What are the coordinates of point $(2, -3)$ after it is reflected over the $x$-axis? | 1) (2, 3)  
2) (-2, 3)  
3) (-2, -3)  
4) (-3, 2) |
| 5. Point $(-2, 3)$ is reflected in the $x$-axis. In which quadrant does its image lie? | 1) I  
2) II  
3) III  
4) IV |
| 6. Reflecting $(5, 1)$ in the $y$-axis yields an image of | 1) (5, -1)  
2) (-5, -1)  
3) (5, 1)  
4) (-5, 1) |
| 7. The image of point $(3, 4)$ when reflected in the $y$-axis is | 1) (-3, -4)  
2) (-3, 4)  
3) (3, -4)  
4) (4, 3) |
| 8. What is the image of the point $(2, -3)$ after the transformation $r_y$? | 1) (2, 3)  
2) (-2, -3)  
3) (-2, 3)  
4) (-3, 2) |
<table>
<thead>
<tr>
<th>Question</th>
<th>Options</th>
</tr>
</thead>
</table>
| 9. What are the coordinates of point $P$, the image of point $(3, -4)$ after a reflection in the line $y = x$? | 1) $(3, 4)$  
2) $(-3, 4)$  
3) $(4, -3)$  
4) $(-4, 3)$ |
| 10. What is the image of $(5, -2)$ under the transformation $r_y = x$?       | 1) $(-5, 2)$  
2) $(5, 2)$  
3) $(2, 5)$  
4) $(-2, 5)$ |
| 11. If the point $(2, -5)$ is reflected in the line $y = x$, then the image is? | 1) $(5, -2)$  
2) $(-2, 5)$  
3) $(-5, 2)$  
4) $(-5, -2)$ |
| 12. The coordinates of point $A$ are $(-3a, 4b)$. If point $A'$ is the image of point $A$ reflected over the line $y = x$, the coordinates of $A'$ are | 1) $(4b, -3a)$  
2) $(3a, 4b)$  
3) $(-3a, -4b)$  
4) $(-4b, -3a)$ |
| 13. A function, $f$, is defined by the set $(2, 3), (4, 7), (-1, 5)$. If $f$ is reflected in the line $y = x$, which point will be in the reflection? | 1) $(5, -1)$  
2) $(-5, 1)$  
3) $(1, -5)$  
4) $(-1, 5)$ |
| 14. What is the image of point $(-3, -1)$ under a reflection in the origin? | 1) $(3, 1)$  
2) $(-3, 1)$  
3) $(1, 3)$  
4) $(-1, -3)$ |
| 15. The point $(-3, -2)$ is reflected in the origin. The coordinates of its image are | 1) $(-2, -3)$  
2) $(3, 2)$  
3) $(2, 3)$  
4) $(-3, 2)$ |
| 16. If $M(-2, 8)$ is reflected in the $y$-axis, what are the coordinates of $M'$, the image of $M$? |  
|
| 17. Find the image of $(1, 5)$ when it is reflected over the line $y = x$. |  
|
| 18. Find the image of $P(2, -5)$ under the transformation $r_y = x$. |  
|
1. The coordinates of the endpoints of AB are A(0, 2) and B(4, 6). Graph and state the coordinates of A' and B', the images of A and B after AB is reflected in the x-axis.

2. Triangle SUN has coordinates S(0,6), U(3,5), and N(3,0). On the accompanying grid, draw and label ΔSUN. Then, graph and state the coordinates of ΔS'U'N', the image of ΔSUN after a reflection in the y-axis.

3. On the accompanying grid, draw and label quadrilateral ABCD with points A(1, 2), B(6, 1), C(7, 6), and D(3, 7). On the same set of axes, plot and label quadrilateral A'B'C'D', the reflection of quadrilateral ABCD in the y-axis. Determine the area, in square units, of quadrilateral A'B'C'D'.

4. On the accompanying set of axes, draw the reflection of ABCD in the y-axis. Label and state the coordinates of the reflected figure.
5. Triangle $\triangle ABC$ has coordinates $A(2, 0)$, $B(1, 7)$, and $C(5, 1)$. On the accompanying set of axes, graph, label, and state the coordinates of $\triangle A'B'C'$, the reflection of $\triangle ABC$ in the $y$-axis.

6. Carson is a decorator. He often sketches his room designs on the coordinate plane. He has graphed a square table on his grid so that its corners are at the coordinates $A(2, 6)$, $B(7, 8)$, $C(9, 3)$ and $D(4, 1)$. To graph a second identical table, he reflects $\square ABCD$ over the $y$-axis. On the accompanying set of coordinate axes, sketch and label $\square ABCD$ and its image $\square A'B'C'D'$, which show the locations of the two tables. Then find the number of square units in the area of $\square ABCD$. 
7. Triangle $ABC$ has vertices $A(-2, 2)$, $B(-1, -3)$, and $C(4, 0)$. Find the coordinates of the vertices of $\triangle A'B'C'$, the image of $\triangle ABC$ after the transformation $r_{x\text{-axis}}$.

8. Triangle $XYZ$, shown in the diagram below, is reflected over the line $x = 2$. State the coordinates of $\triangle X'Y'Z'$, the image of $\triangle XYZ$.

9. Two parabolic arches are to be built. The equation of the first arch can be expressed as $y = -x^2 + 9$, with a range of $0 \leq y \leq 9$, and the second arch is created by the transformation
T. On the accompanying set of axes, graph the equations of the two arches. Graph the line of symmetry formed by the parabola and its transformation and label it with the proper equation.

10. In the accompanying diagram of circle O, diameter AB is perpendicular to chord CD at point E. What is the image of AC in AB?

1) AD
2) BD
3) ED
4) AE

11. The parabola shown in the accompanying diagram undergoes a reflection in the y-axis. What will be the coordinates of the turning point after the reflection?

1) (3, -1)
12. The accompanying graph shows the relationship between kinetic energy, $y$, and velocity, $x$.

![Graph showing the relationship between kinetic energy and velocity]

The reflection of this graph in the line $y = x$ is

1) ![Reflection 1]
2) ![Reflection 2]
3) ![Reflection 3]
4) ![Reflection 4]

1. What is the image of the point (-5, 2) under the translation $T_{3,-4}$?
   1) (-9, 5)
   2) (-8, 6)
   3) (-2, -2)
   4) (-15, -8)

2. A translation moves $P(3, 5)$ to $P1(6, 1)$. What are the coordinates of the image of point (-3, -5) under the same translation?
   1) (0, -9)
   2) (-5, -3)
   3) (-6, -1)
   4) (-6, -9)

3. The image of point (-2, 3) under translation $T$ is (3, -1). What is the image of point (4, 2) under the same translation?
   1) (-1, 6)

4. The image of the origin under a certain translation is the point (2, -6). The image of point (-3, -2) under the same translation is the point
   1) (-6, 12)
2) (0, 7)  3) (5, 4)  4) (9, -2)

2) (-5, 4)  3) \(-\frac{3}{2}, \frac{1}{3}\)  4) (-1, -8)

5. Triangle ABC has vertices A(1, 3), B(0, 1), and C(4, 0). Under a translation, A~, the image point of A, is located at (4, 4). Under this same translation, point C~ is located at:
1) (7, 1) \((x + 3, y + 1)\)  ANS. 1
2) (5, 3)
3) (3, 2)
4) (1, -1)

6. A design was constructed by using two rectangles ABDC and A'B'C'D'. Rectangle A'B'C'D' is the result of a translation of rectangle ABDC. The table of translations is shown below. Find the coordinates of points B and D'.

<table>
<thead>
<tr>
<th>Rect. ABCD</th>
<th>Rect. A'B'C'D'</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (2, 4)</td>
<td>A' (3, 1)</td>
</tr>
<tr>
<td>B</td>
<td>B' (-5, 1)</td>
</tr>
<tr>
<td>C (2, -1)</td>
<td>C' (3, -4)</td>
</tr>
<tr>
<td>D (-6, -1)</td>
<td>D'</td>
</tr>
</tbody>
</table>

7. Triangle TAP has coordinates T(-1, 4), A(2, 4), and P(2, 0). On the set of axes below, graph and label ΔT'A'P', the image of ΔTAP after the translation \((x, y) (x - 5, y - 1)\).
3. What is the image of point (2, 4) under the translation $T_{-6,1}$?

   1) (-4, 3)  2) (-4, 5)  3) (8, 3)  4) (8, 5)

4. If translation $T$ maps point (-3, 1) onto point $A'(5, 5)$, which is translation $T$?

   1) $T_{2,4}$  2) $T_{2,6}$  3) $T_{8,6}$  4) $T_{8,4}$

5. What is the image of (-2, 3) after the transformation $T_{3,-1}$?

6. What is the image of the point (-3, 4) under the translation $T_{-2,0}$?

7. Find the coordinates of the image of (-3, 4) under the transformation $T_{-2,3}$.

8. Find the coordinates of $P'$, the image of $P(-3, 4)$ under the translation $T_{4,1}$.

9. If the transformation $T_{x,y}$ maps point $A(1, -3)$ onto point $A'(-4, 8)$, what is the value of $x$?

10. Translation $T$ maps point (2, 6) to point (4, -1). What is the image of point (-1, 3) under translation $T$?

11. A translation maps $P(3, -2)$ to $P'(1, 1)$. Under the same translation, find the coordinates of $Q'$, the image of $Q(-3, 2)$.

12. A translation maps $P(4, 1)$ to $P'(2, -1)$. What are the coordinates of $Q'$, the image of $Q(1, 3)$ under the same translation?

13. A translation maps $P(4, -3)$ onto $P'(0, 0)$. Find the coordinates of $Q'$, the image of $Q(2, 1)$ under the same translation.

14. A translation maps the origin to the point (5, -3). What is the image of the point (-3, 2) under the same translation?

15. Under a given translation, the origin maps onto the point (3, 5). What is the image of the point (7, -1) under the same translation?

16. A translation maps the point (5, -2) to a point (0, -2). What is the image of point (0, -2) under the same translation?
17. A translation maps (2, 1) onto (-3, 2). Find the image of (4, -1) under the same translation.

18. A translation maps P(3, -2) onto P'(5, 0). Find the coordinates of the image of Q(4, -6) under the same translation.

1. The point (3, -2) is rotated 90° about the origin and then dilated by a scale factor of 4. What are the coordinates of the resulting image?

   1) (-12, 8)
   2) (12, -8)
   3) (8, 12)
   4) (-8, -12)

2. What is the image of point A(4, 2) after the composition of transformations defined by $R_{90} \circ r_y = x$?

   1) (-4, 2)
   2) (4, -2)
   3) (-4, -2)
   4) (2, -4)

3. What is the image of point (1, 1) under $r_{x-axis} \circ R_{90}$?

   1) (1, 1)
   2) (1, -1)
   3) (-1, 1)
   4) (-1, -1)

4. What are the coordinates of point A', the image of point A(-4, 1) after the composite transformation $R_{90} \circ r_y = x$ where the origin is the center of rotation?

   1) (-1, -4)
   2) (-4, -1)
   3) (1, 4)
   4) (4, 1)

5. The coordinates of $\triangle JRB$ are J(1, -2), R(-3, 6), and B(4, 5). What are the coordinates of the vertices of its image after the transformation $T_{2,-1} \circ r_{y-axis}$?

   1) (3, 1), (-1, -7), (6, -6)

6. If the coordinates of point P are (2, -3), then $R_{90} \circ R_{180}$ is

   1) (-2, 3)
   2) (-2, -3)
   3) (3, -2)
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2)</td>
<td>(3, -3), (-1, 5), (6, 4)</td>
<td>4)</td>
</tr>
<tr>
<td>3)</td>
<td>(1, -3), (5, 5), (-2, 4)</td>
<td></td>
</tr>
<tr>
<td>4)</td>
<td>(-1, -2), (3, 6), (-4, 5)</td>
<td></td>
</tr>
</tbody>
</table>

7. Find the coordinates of $A'$ if the coordinates of $A$ are $(6, 1)$.

8. Find the coordinates of the image of $(2, 4)$ under the transformation $r_y$ followed by $T_{3, -5}$.

9. What is the image that results from this composition of transformations $r_x$ followed by $R_{90}$ of $(-3, 0)$?

10. Find the coordinates of point $N(-1, 3)$ under the composite $r_y$ followed by $T_{-2, 4}$.

11. If the coordinates of $A$ are $(2, 3)$, what are the coordinates of $A'$, the image of $A$ after $R_{90}$ followed by $r_y$?

12. If the coordinates of $B$ are $(1, -5)$, what are the coordinates of $B'$, the image of $B$ after $R_{90}$ followed by $r_x$?

13. Find the image of point $A(3, -2)$ under the composition of translations $T_{2, 1} \circ T_{-6, -4}$.

14. Which transformation is equivalent to the composite line reflections $r_y \circ r_{y = x}$?
   1) a rotation
   2) a dilation
   3) a translation
   4) a glide reflection

15. Write a single translation that is equivalent to $T_{3, -1}$ followed by $T_{-5, 5}$.
1. The accompanying graph represents the figure 1.

Which graph represents 1 after a transformation defined by \( r_y = x \circ R_{90} \)?

1)  
2)  
3)  
4)  

2. Given point A(-2, 3). State the coordinates of the image of A under the composition \( T_{-3, -4} \circ r_{x-axis} \).

3. On the accompanying grid, graph and label AB, where A is (0, 5) and B is (2, 0). Under the transformation \( r_{y-axis} \circ r_{x-axis} \), A maps to A`` and B maps to B``. Graph and label A``B``. What single transformation would map
4. On the accompanying grid, graph and label \( \Delta ABC \) with vertices \( A(3, 1), B(0, 4), \) and \( C(-5, 3) \). On the same grid, graph and label \( \Delta A'B'C' \), the image of \( \Delta ABC \) after the transformation \( r_{x \text{-axis}} \circ r_{y=x} \).
5. The coordinates of the vertices of \( \triangle ABC \) are \( A(1, 6), B(2, 9), \) and \( C(7, 10) \).

a) On the graph below, draw and label \( \triangle ABC \).

b) Graph and state the coordinates of \( \triangle A'B'C' \), the image of \( \triangle ABC \) after a reflection over the line \( y = x \).

c) Graph and state the coordinates of \( \triangle A''B''C'' \), the image of \( \triangle A'B'C' \) after a reflection in the \( x \)-axis.

d) Graph and state the coordinates of \( \triangle A'''B'''C''' \), the image of \( \triangle A''B''C'' \) after the transformation \((x, y) \rightarrow (x - 5, y + 3)\).

---

6. Given: \( \triangle ABC \) with coordinates \( A(1, 2), B(0, 5), \) and \( C(5, 4) \).

a) On the graph below, draw and label \( \triangle ABC \).

b) Graph and state the coordinates of \( \triangle A'B'C' \), the image of \( \triangle ABC \) after the translation \( T_{-6, 3} \).

c) Graph and state the coordinates of \( \triangle A''B''C'' \), the image of \( \triangle A'B'C' \) after a reflection in the \( x \)-axis.

d) Graph and state the coordinates of \( \triangle A'''B'''C''' \), the image of \( \triangle A''B''C'' \) after a reflection in the origin.
7. Triangle \( ABC \) has coordinates \( A(-1, 3), B(3, 7), \) and \( C(0, 6). \)

a) On the graph below, draw and label \( \triangle ABC \).

b) Graph and state the coordinates of \( \triangle A'B'C' \), the image of \( \triangle ABC \) after a reflection in the line \( y = x \).

c) Graph and state the coordinates of \( \triangle A''B''C'' \), the image of \( \triangle A'B'C' \) following \( r_{y-axis} \).

d) Graph and state the coordinates of \( \triangle A'''B'''C''' \), the image of \( \triangle A''B''C'' \), after a translation that maps \( P(0, 0) \) onto \( P'(0, -5) \).

8. Triangle \( ABC \) has coordinates \( A(-1, 2), B(6, 2), \) and \( C(3, 4). \)

a) On the grid below, draw and label \( \triangle ABC \).

b) Graph and state the coordinates of \( \triangle A'B'C' \), the image of \( \triangle ABC \) after the composition \( \triangle ABC \).
c) Write a transformation equivalent to $R_{90} \circ r_{x-axis}$.

9. On the graph below, draw and label $\Delta PQR$, whose vertices are $P(3, 5)$, $Q(9, 5)$, and $R(7, 7)$. On the same set of axes, graph and state the coordinates of

a) $\Delta P'Q'R'$, the image of $\Delta PQR$ after $R_{90}$.

b) $\Delta P''Q''R''$, the image of $\Delta P'Q'R'$ after $r_{x-axis}$.

c) $\Delta P'''Q'''R'''$, the image of $\Delta P''Q''R''$ after $r_{y-axis}$.

Based upon these graphs, write a single transformation that shows the composition $r_{y-axis} \circ r_{x-axis} \circ R_{90}$.
0. Given $\triangle ABC$ with points $A(4, 3)$, $B(4, -2)$, and $C(2, 3)$. On the grid below, sketch $\triangle ABC$. On the same set of axes, graph and state the coordinates of $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection in the line $y = x$. On the same set of axes, graph and state the coordinates of $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after the translation $T_{-4, 3}$.

11. The coordinates of the vertices of parallelogram $ABCD$ are $A(-2, 2)$, $B(3, 5)$, $C(4, 2)$, and $D(-1, -1)$. State the coordinates of the vertices of parallelogram $A''B''C''D''$ that result from the transformation $r_{y-axis} \circ T_{2, -3}$. 

12. Given triangle $ABC$ with coordinates $A(-1, -2)$, $B(0, -4)$, and $C(3, -1)$.

   a) On the graph below, draw and label $\triangle ABC$.

   b) Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after translation $T_{4, -3}$.

   c) Graph and label $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after a reflection in the origin.

   d) Graph and label $\triangle A'''B'''C'''$, the image of $\triangle A''B''C''$ after a reflection in the line $y = -x$. 
13. Triangle $ABC$ has coordinates $A(-3, -7)$, $B(-3, -3)$, and $C(0, -3)$.

a) On the graph below, graph and label $\triangle ABC$.

b) Graph and state the coordinates of $\triangle A'B'C'$, the image of $\triangle ABC$ after a point reflection in the origin.

c) Graph and state the coordinates of $\triangle A''B''C''$, the image of $\triangle A'B'C'$ reflected in the line $y = 2$.

d) Graph and state the coordinates of $\triangle A'''B'''C'''$, the image of $\triangle A''B''C''$ after translation $T_{-8, 2}$.

14. a) On the accompanying grid, graph the equation $2y = 2x^2 - 4$ in the
interval \( -3 \leq x \leq -3 \) and label it a.

b) On the same grid, sketch the image of a under \( T_{5,2} \circ r_{\text{x-axis}} \) and label it b.

---

1. Triangle \( ABC \) has coordinates A(1, 1), B(5, 1), and C(4, 3). Given the transformations \( T, U, \) and \( W \) described below:

   \[ T: (x, y) \rightarrow (x, -y) \quad U: (x, y) \rightarrow (x - 6, y + 6) \quad W: (x, y) \rightarrow (-2x, -2y) \]

   a) Graph \( \Delta ABC \) and graph and state the coordinates of its image \( \Delta A'B'C' \), after transformation \( T \).

   b) Graph and state the coordinates of \( \Delta A''B''C'' \), the image of \( \Delta ABC \) after transformation \( U \).

   c) Graph and state the coordinates of \( \Delta A'''B'''C''' \), the image of \( \Delta ABC \) after transformation \( W \).

   d) Which transformation, \( T, U, \) or \( W \), is not an isometry?

   e) Which transformation, \( T, U, \) or \( W \), does not preserve orientation?
2. Triangle ABC has vertices A(2, -2), B(5, -2), and C(3, -4).

   a) On the set of axes below, graph and label ΔABC and its image under each of the following transformations. State the coordinates of the vertices for each image of ΔABC.

      T: \( (x, y) \rightarrow (-x, y) \)  
      U: \( (x, y) \rightarrow (x - 4, y + 4) \)  
      W: \( (x, y) \rightarrow (-2x, -2y) \)

   b) Which transformation, T, U, or W, is not an isometry?

   c) Which transformation, T, U, or W, does not preserve orientation?